TI-68k vs HP49G+ Benchmark

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Notes

- Like other computer algebra benchmarks, the CAS timings include evaluation time but not display time

- The graphics performance benchmark includes computation time as well as rendering time

- The display-routine performance benchmark includes conversion/printing time but not evaluation time

- Free add-on programs are included in this benchmark (need to include ASI and add references to add-ons used)

- Performance comparisons are done only when both machines have that particular functionality

- For the most part, this benchmark is currently focused on math problems, not applications such as engineering

- Inputs are normally given in TI syntax

- For floating point arithmetic, approximate values of the arguments shown are used

- The Laurent series capability on the 68k currently returns only approximate numeric results

- Time is given in seconds, unless otherwise noted

- Timing for the 68k was done on my Voyage 200:

  AMS 2.09, 168 KB free RAM, empty history area, no TSRs or kernels installed

  Mode settings: DisplayDigits $\rightarrow$ Float12, Angle $\rightarrow$ Radian, ExponentialFormat $\rightarrow$ Normal, ComplexFormat $\rightarrow$ Rectangular, PrettyPrint $\rightarrow$ On, Exact/Approx $\rightarrow$ Auto.

- HP49G+:

  ROM revision 1.23, CAS version 4.20031005

  Flag settings: {#8205010FF0h, #0h, #880404402000000h, #0h}

- I think this might be a good chance to make a set of regression tests (the same as or similar to the examples below)

- A few examples (e.g. for polynomial factorization) may have been from Bernard Parisse; add references if needed

- Disclaimer: Keep in mind that it is significantly harder to develop a fair benchmark for symbolics than for numerics. The examples give only a rough idea of the strengths and weaknesses. Also, I do not know many of the methods used internally by the CAS's.

Performance

Add to table:

Special functions (Airy, Bernoulli/Euler numbers, Fibonacci, Fresnel, erf, exponential integrals, Hurwitz zeta, $\frac{\partial \zeta(s,a)}{\partial s}$, polylogs, hypergeometrics, orthogonal polynomials, cyclotomic, number theoretical), construction of special matrices (Hilbert, Vandermonde)

Things to try:

Poles, essential singularities, branch cuts, and discontinuous functions for symbolic and numeric definite integration. High orders for differentiation. Non-differentiable functions and discontinuous functions for optimization. Non-squarefree polynomials and large coefficients for factorization. Finite (abs, sign, min, max, when) and infinite (floor, ceiling, round, fpart, mod, intdiv) piecewise functions for integration and equation solving. Integrands of type $R\left(x, \sqrt{x^2 + b \cdot x + c}\right)$.

Consider including non-doable ("give-up") examples.

Problem	TI	HP	Comments
$91\,356\,200\,129 + 7\,868\,039\,544$	0.001	☐	Integer arithmetic $-$ addition
$356\,530\,692 * 200\,980\,515\,627$	0.002	☐	Integer arithmetic $-$ multiplication
7^{477}	0.17	☐	Integer arithmetic $-$ exponentiation
$\sqrt{3} + \sqrt{5}$	☐	☐	Real floating point arithmetic $-$ addition
$\sqrt{3} * \sqrt{5}$	☐	☐	Real floating point arithmetic $-$ multiplication
$\sqrt{3} / \sqrt{5}$	☐	☐	Real floating point arithmetic $-$ division
$\left(\sqrt{3}\right)^{\sqrt{5}}$	☐	☐	Real floating point arithmetic $-$ exponentiation
$\sqrt{13}$	☐	☐	Real floating point arithmetic $-$ square root
$\cos(\pi + e)$	☐	☐	Real floating point arithmetic $-$ cosine
☐	☐	☐	Real floating point arithmetic $-$ arctangent
☐	☐	☐	Complex floating point arithmetic $-$ addition
☐	☐	☐	Complex floating point arithmetic $-$ multiplication
☐	☐	☐	Complex floating point arithmetic $-$ division
☐	☐	☐	Complex floating point arithmetic $-$ exponentiation
☐	☐	☐	Complex floating point arithmetic $-$ square root
☐	☐	☐	Complex floating point arithmetic $-$ absolute value
☐	☐	☐	Complex floating point arithmetic $-$ natural logarithm
$\gcd\left(29^{400},\ 31^{300}\right)$	11.42	7.23	Integer gcd
$180!$	0.29	0.51	Factorial

	TI-68k	HP49G+	
$\text{isPrime}\left(2^{127} - 1\right)$	34.95	32.42	Primality testing
factor (211!)	5.21	7.10	Integer factorization
factor (4 498 414 682 539 051)	5.74	3.50	Integer factorization with moderately large factors
factor (num1)	72.16	□	Integer factorization with large factors
$\text{expand}\left((x + y)^{99}\right)$	6.47	Hang	Polynomial expansion
$\text{expand}\left(1 / \left(x^6 + 1\right), x\right)$	0.88	4.09	Partial fraction expansion
expand (1 / poly5)	12.32	□	Partial fraction expansion
SqrFree (poly6, x)	2.05	□	Squarefree factorization
$\text{factor}\left(x^{100} - 1\right)$	1.09	□	Univariate factorization
$\text{factor}\left(x^{100} - 1, x\right)$	7.26	Hang	Univariate factorization
factor (poly1)	□	□	Univariate factorization
□	□	□	Univariate factorization with parameters
factor (poly2)	□	□	Multivariate factorization
factor (ratfunc1)	□	□	Multivariate factorization
solve (3 x + 9 = 0, x)	0.06	□	Univariate polynomial solving
$\text{solve}\left(x^6 - x^4 - 4x^2 + 4 = 0, x\right)$	1.37	□	Univariate polynomial solving − real solutions
$\text{cSolve}\left(x^6 - x^4 - 4x^2 + 4 = 0, x\right)$	1.63	□	Univariate polynomial solving − complex solutions
$\text{cZeros}\left(x^4 + x^3 + x^2 + x + 1, x\right)$	9.05	□	Univariate polynomial solving − complex solutions
$\text{cZeros}\left(x^6 - 1, x\right)$	0.18	2.17	Univariate polynomial solving − complex solutions
solve $((x - 7) \cdot (x - 5) \cdot (x - 3) = 0, x) \mid x \leq 6$	□	□	Univariate polynomial solving − constrained
cSolve $(x\_^5 = x\_ \text{ and imag}(x\_) \neq 0, x\_)$	□	□	Univariate polynomial solving − constrained
□	□	□	Univariate polynomial solving − parameters
$\text{solve}\left(x^2 + y^2 = 1, \{x, y\}\right)$	0.42	□	Multivariate polynomial solving − real solutions
$\text{zeros}\left(\{x^3 + 3 \cdot x \cdot y + y^3, x + y^3\}, \{x, y\}\right)$	7.19	□	Multivariate polynomial solving − real solutions
$\text{cSolve}\left(x^2 + y^2 = 1, \{x, y\}\right)$	0.25	□	Multivariate polynomial solving − complex solutions
$\text{cZeros}\left(\{x^3 + 3 \cdot x \cdot y + y^3, x + y^3\}, \{x, y\}\right)$	9.05	□	Multivariate polynomial solving − complex solutions
solve (polysys, {x, y})	□	□	Polynomial system solving − real solutions
cSolve (polysys, {x, y})	□	□	Polynomial system solving − complex solutions
□	□	□	Univariate polynomial solving (numeric)
PolyGCD (poly7, poly8)	0.28	□	Univariate polynomial gcd
□	□	□	Multivariate polynomial gcd
□	□	□	Univariate polynomial reduction modulo a prime
□	□	□	Polynomial reduction modulo a polynomial
□	□	□	Univariate polynomial quotient and remainder
$d\left(x^{71}, x, 60\right)$	0.30	□	Differentiation − polynomials
$d\left(5 \cdot x^{11} + 3 \cdot x^7 - 7 \cdot x^3 + 21 \cdot x^2 - 1, x, 5\right)$	0.12	□	Differentiation − polynomials
□	□	□	Differentiation − rational functions
$d\left(\sqrt{x}, x, 5\right)$	0.05	2.05	Differentiation − algebraic functions $(\mathcal{R}3)$
$d\left(e^{a*x}, x, 100\right)$	0.82	□	Differentiation − transcendental functions

$d(\cos(\ln(x)), x, 50)$	3.32	□	Differentiation − compositions of functions
$d(x^{\sin(x)}, x, 3)$	4.63	□	Differentiation − compositions of functions
$d(f(x)/g(x), x, 4)$	11.47	□	Differentiation − undefined functions
$\lim(\sin(\sqrt{x}), x, 0)$	0.01	□	Limit
$\lim((1 + 1/x)^x, x, \infty)$	0.12	4.93	Limit
$\lim(x^{12} \cdot e^{-x}, x, \infty)$	0.25	5.35	Limit
$\lim((x^4 - 6x^3 + x^2 + 3)/(x-1), x, 1)$	0.16	□	Limit − pole
$\lim(\sin(1/x), x, 0)$	0.04	2.31	Limit − essential singularity
$\lim(\ln(x)/\sqrt{x}, x, \infty)$	0.08	6.36	Limit
$\lim(\sqrt{x+a} - \sqrt{x}, x, \infty)$	0.84	22.44	Limit
$\lim(x^{22}/77^x, x, \infty)$	0.72	11.01	Limit
$\lim((3^x + 5^x)^{1/x}, x, \infty)$	0.37	59.51	Limit
$\text{taylor}(x^3 + 2x, x, 3, 2)$	0.07	□	Taylor series − polynomials (about 2 to order 3)
$\text{taylor}(\tan(x), x, 10)$	13.00	□	Taylor series
$\text{taylor}(\ln(x+1), x, 20)$	0.63	□	Taylor series
$\text{taylor}(\sin(\cos(x)), x, 8, 2)$	12.47	□	Taylor series
$\text{taylor}\left(1/\sqrt{1 - v^2/c^2}, v, 8\right)$	9.98	□	Taylor series
□	□	□	Taylor series
$\text{nSeries}(\sin(x)/(e^x - 1), x, 0, 3)$	□	□	Laurent series (about zero to order 3)
$\text{nSeries}(e^{1/x}, x, \infty, 5)$	□	□	Asymptotic series
□	□	□	Indefinite integration − polynomials
$\int(\sin(x) \cdot \cos^n(x))\,dx$	0.38	□	Indefinite integration − $f(x) \cdot f'(x)$
$\int\left(\cos(\ln(x))/\left(x \cdot \sqrt{\sin(\ln(x))}\right)\right)dx$	0.14	2.80	Indefinite integration − $f'(x)/f(x)$
$\int(1/(x^8 + 1))\,dx$	19.26	□	Indefinite integration − rational functions
$\int\left(\frac{x^2+1}{x^4+x^2+1}\right)dx$	1.46	1.78	Indefinite integration − rational functions
$\text{Horowitz}(\text{poly3}, (x^7 - x + 1)^2, x)$	□	2.09	Indefinite integration − rational functions
$\int(x^2 \cdot (a \cdot x^3 + b \cdot x^2)^{1/3})\,dx$	□	□	Indefinite integration − algebraic functions
$\int(x^5/(4x^2 + 9)^{1/3})\,dx$	0.87	23.49	Indefinite integration − algebraic functions
$\int(x/\sqrt{a \cdot x^2 + b \cdot x + c})\,dx$	1.04	15.53	Indefinite integration − algebraic functions
$\int(1/(\sqrt{x} + x^{1/3}))\,dx$	0.53	5.99	Indefinite integration − algebraic functions
$\int(x/\sqrt{1 - x^4})\,dx$	0.23	□	Indefinite integration − algebraic functions
$\int(1 - \sqrt{x})^{p-1}\,dx$	1.02	□	Indefinite integration − algebraic functions (G & R-based)

$\int \left(\frac{1}{\sqrt{x^2+1}} + \frac{100}{\sqrt{x^2+10\,000}} \right) dx$	0.23	7.09	Indefinite integration − algebraic functions $(\mathcal{R}4)$
$\int \left(\sin(a \cdot x + b) \cdot \sin(x)^5 \right) dx$	□	□	Indefinite integration − transcendental functions
$\int \left(\sin(x)^{12} \cdot \cos(x)^{29} \right) dx$	□	□	Indefinite integration − transcendental functions
$\int \left(e^x \cdot \sin^2(x) \right) dx$	□	□	Indefinite integration − transcendental functions
$\int \left(\ln(x)^4 / x^m \right) dx$	□	□	Indefinite integration − transcendental functions
$\int (x \cdot a^x \cdot \cos(x)) dx$	□	□	Indefinite integration − transcendental functions
$\int \left(x \cdot \sinh^{-1}(a/x) \right) dx$	□	□	Indefinite integration − transcendental functions
$\int \left(\tan^{-1}\left(\frac{x-5}{x-3} \right) \right) dx$	□	□	Indefinite integration − transcendental functions $(\mathcal{R}1)$
$\int \left(f(x)^{g(x)} \cdot \left(\frac{f'(x) \cdot g(x)}{f(x)} + \ln(f(x)) \cdot g'(x) \right) \right) dx$	□	□	Indefinite integration − undefined functions
□	□	□	Definite integration − polynomials
$\int_{a-1}^{a+1} \frac{1}{(x-a)^{999}} dx$	0.29	Hang	Definite integration − rational functions
$\int_1^\infty \frac{1}{z^n} dz \,\big|\, n > 1$	□	□	Definite integration − rational functions
□	□	□	Definite integration − rational functions
$\int_{-r}^{r} \sqrt{x^2 - r^2} \, dx$	0.84	4.45	Definite integration − algebraic functions
$\int_0^1 \left(1 - \sqrt{x} \right)^{p-1} dx \,\big|\, p > 0$	□	□	Definite integration − algebraic functions $(\mathcal{R}5)$
$\int_{-\infty}^{a} x \cdot e^x \, dx$	0.32	5.35	Definite integration − transcendental functions
$\int_0^{\pi/4} (\tan^5(x)) \, dx$	62.47	4.70	Definite integration − transcendental functions $(\mathcal{R}5)$
$\int_0^{\pi} \frac{\sin^3(x)}{p+q\cos(x)} \, dx$	3.63	17.93	Definite integration − transcendental functions $(\mathcal{R}5)$
$\int_0^\infty t^2 \cdot e^{-s*t} \, dt \,\big|\, s > 0$	1.47	□	Definite integration − transcendental functions
$\int_{-2}^{2} (\text{abs}(\cos(z))) \, dz$	12.43	□	Definite integration − piecewise functions
□	□	□	Definite integration − piecewise functions
$\int_0^\infty (f(t)/t) \, dt$	0.16	□	Definite integration − undefined functions
$\int_{-\infty}^{\mu} \left(x \cdot e^{(x-\mu)/\sigma} / (2\,\sigma) \right) dx \,\big|\, \sigma > 0$	□	□	Definite integration − other improper integrals
$\int_0^\infty \left(x \cdot e^{-x^2} \right) dx$	0.43	1.52	Definite integration − other improper integrals
$\int_0^\infty \frac{(1+x)^{p-1}}{(x+a)^{p+1}} \, dx \,\big|\, a > 0$	□	□	Definite integration − other improper integrals $(\mathcal{R}5)$
$\int_0^\infty \text{sech}(a \cdot x) \, dx \,\big|\, a > 0$	□	□	Definite integration − other improper integrals $(\mathcal{R}5)$
$\int_{-\infty}^{\infty} \frac{x^2+1}{x^4+x^2+1} \, dx$	2.94	6.91	Definite integration − contour-type improper integrals
$\text{CPVInt}\left(1/(5x^4 + 6x), x, -1, 1 \right)$	5.87	3.67	Definite integration − principal value integrals
$\text{CPVInt}(\tan(x), x, \pi/4, 3\pi/4)$	4.38	3.83	Definite integration − principal value integrals
$\text{CPVInt}(1/(x \cdot \ln(x)), x, 1/2, 2)$	5.60	3.16	Definite integration − principal value integrals

$\int_{-1}^{1}\int_{-1}^{1} 1 \, dx \, dy$	0.02	□	Definite integration − multidimensional
$\int_{0}^{4}\int_{0}^{2*\sqrt{z}}\int_{0}^{\sqrt{4z-y^2}} 1 \, dx \, dy \, dz$	0.76	□	Definite integration − multidimensional
$\int_{0}^{1}\int_{0}^{x}(x^2 + y^2) \, dy \, dx$	0.11	□	Definite integration − multidimensional
$\text{nInt}\left(x^2 - 2x + 3, x, 0, 1\right)$	□	□	Numeric integration − polynomials
$\text{nInt}\left(1/\left(x^4 + x^2 + 9/10\right), x, -1, 1\right)$	□	□	Numeric integration − rational functions
$\text{nInt}\left(\sec(\tan(x)), x, 0, 1\right)$	□	□	Numeric integration − finite intervals
$\text{nInt}\left(\ln(x), x, -1, 1\right)$	□	□	Numeric integration − finite intervals
$\text{nInt}\left(\cos(x), x, -10, 10\right)$	□	□	Numeric integration − even integrands
$\text{nInt}\left(\text{when}(x \geq 0 \text{ and } x < 0.3, 0, 1), x, 0, 1\right)$	□	□	Numeric integration − piecewise integrands
$\text{nInt}\left(\min(\sin(x), \cos(x)), x, -3\pi/2, 3\pi/2\right)$	□	□	Numeric integration − piecewise integrands
$\text{nInt}\left(1/\sqrt{|x|}, x, -1, 1\right)$	□	□	Numeric integration − singularity
$\text{nInt}\left(1/x^8, x, -\pi, \pi\right)$	□	□	Numeric integration − singularity
$\text{nInt}\left(\frac{x\sin(x)}{1-\cos(x)}, x, 0, \pi/2\right)$	□	□	Numeric integration − singularity at endpoint (R5)
$\text{nInt}\left(\sqrt{(x+1)\cdot(x-1)}\cdot\ln(x), x, 0, 1\right)$	□	□	Numeric integration − singularity at endpoint
$\text{nInt}\left(x^3\cdot\ln\left(\left|\left(x^2-1\right)\left(x^2-2\right)\right|\right), x, 0, 3\right)$	□	□	Numeric integration − singularity
$\text{nInt}\left(\sin(x)/\left(x^{100}+1\right), x, 1, \infty\right)$	□	□	Numeric integration − oscillatory integrands
$\text{nInt}\left(\frac{1}{10\sqrt{2\pi}}\cdot e^{-\frac{(x-20)^2}{2*10^2}}, x, 104, 136\right)$	7.51	□	Numeric integration − Gaussians
$\text{nInt}\left(1/\sqrt{x}, x, 0, \infty\right)$	□	□	Numeric integration − other improper integrals
$\text{nInt}\left(\ln(x)/\left(10\cdot x^2+1\right), x, 0, \infty\right)$	□	□	Numeric integration − other improper integrals
$\text{nInt}\left(\text{nInt}\left(\text{abs}(x-y)^{3/2}, x, 0, 1\right), y, 0, 1\right)$	□	□	Numeric integration − multidimensional
$\text{Laplace}\left(t^7\cdot\sin(t), t\right)$	□	□	Laplace transform
□	□	□	Linear constant coefficient ODE
$\text{deSolve}\left(y' = x\cdot y^2, x, y\right)$	□	□	Linear first-order variable coefficient ODE
$\text{deSolve}\left(y' - x\cdot y = 1, x, y\right)$	□	□	Linear first-order variable coefficient ODE
$\text{deSolve}\left(x^2\cdot y'' + a\cdot x\cdot y' = \ln(x), x, y\right)$	□	□	Linear second-order variable coefficient ODE
$\text{deSolve}\left(y\cdot y' = 1, x, y\right)$	□	□	Nonlinear first-order ODE
$\text{deSolve}\left(x^5\cdot y' + y^2 - 4x^4\cdot y + x^8 = 0, x, y\right)$	□	□	Nonlinear first-order ODE (Riccati-type)
$\text{deSolve}\left(\text{ode1}, x, y\right)$	□	□	Nonlinear first-order ODE (Kamke 1.505)
$\text{deSolve}\left(y'' + y\cdot(y')^3 = 0, x, y\right)$	□	□	Nonlinear second-order ODE
$\text{deSolve}\left(8\cdot y'' + 9\cdot(y')^4 = 0, x, y\right)$	□	□	Nonlinear second-order ODE
$\text{deSolve}\left(y'' = a\cdot\sqrt{(y')^2+1}, x, y\right)$	□	□	Nonlinear second-order ODE
$\text{deSolve}\left(y'' = e^y, x, y\right)$	□	□	Nonlinear second-order ODE
$\text{deSolve}\left(y'' = a\cdot y' + y \text{ and initcond1}, x, y\right)$	□	□	Initial value problem
$\text{deSolve}\left(y'' = -y - x \text{ and boundcond1}, x, y\right)$	□	□	Boundary value problem
□	□	□	Numerical ODE solving with initial conditions
$\text{solve}\left(x^{1/3} + \sqrt{x} = 1, x\right)$	□	□	Algebraic equation solving

solve $\left(\ln \left(\sqrt{x} \right) = \sqrt{\ln(x)}, \ x \right)$	☐	☐	Transcendental equation solving
solve $(2 \cdot \sin(x) \cdot \cos(x) = 3, \ x)$	☐	☐	Transcendental equation solving $-$ real solutions
solve $\left(e^{\sin(x)} = 2, \ x \right)$	☐	☐	Transcendental equation solving $-$ real solutions
solve $\left(e^{\sin(x)} = 2, \ x = 30 \right)$	☐	☐	Transcendental equation solving $-$ single real solution
cSolve $(2 \cdot \sin(x) \cdot \cos(x) = 3, \ x)$	☐	☐	Transcendental equation solving $-$ complex solutions
cSolve $\left(e^{\sin(x)} = 2, \ x \right)$	☐	☐	Transcendental equation solving $-$ complex solutions
cSolve $\left(e^{\sin(x)} = 2, \ x = -3 \cdot i \right)$	☐	☐	Transcendental equation solving $-$ single complex solution
solve $\left(\cosh^{-1}\left(z^2 - a \cdot z \right) = a, \ z \right)$	☐	☐	Transcendental equation solving $-$ parameters
Resultnt $\left(x^3 - x \cdot y + z, \ 3\,x^2 - y, \ x \right)$	0.81	0.54	Univariate resultant
fMin $\left(x^2 - 3\,x + 6, \ x \right)$	☐	☐	Univariate function minimization
fMax $(x + 2 \cdot \sin(x), \ x) \mid x \geq -10 \text{ and } x \leq 10$	☐	☐	Univariate function maximization
$\sum \left(k^{10}, \ k, \ 0, \ n \right)$	☐	☐	Indefinite summation $-$ polynomial
$\sum \left(2\,k^7 + 3\,k^5 - k - 1, \ k, \ 0, \ n \right)$	☐	☐	Indefinite summation $-$ polynomial
☐	☐	☐	Indefinite summation $-$ rational function
$\sum \left((-1)^k \cdot k \big/ \left(4\,k^2 - 1 \right), \ k, \ 1, \ n \right)$	☐	☐	Indefinite summation $-$ alternating rational function
$\sum \left(a^{k/2+1/4} \cdot \text{poly4}, \ k, \ 1, \ n \right)$	☐	☐	Indefinite summation $-$ exponential $*$ polynomial
Gosper $(k \cdot k!, \ k, \ 0, \ n)$	☐	☐	Indefinite summation $-$ hypergeometric
Gosper $\left(4^k \big/ \text{nCr}\,(2\,k, \ k), \ k, \ 0, \ n-1 \right)$	☐	☐	Indefinite summation $-$ hypergeometric
$\sum (1/i, \ i, \ 1, \ 400)$	5.42	☐	Summation over a finite range
$\sum (1.0/i, \ i, \ 1, \ 400)$	2.32	☐	Summation over a finite range (approximate)
$\sum \left(1 \big/ k^{42}, \ k, \ 1, \ \infty \right)$	4.42	☐	Summation over an infinite range $-$ Riemann zeta function
☐	☐	☐	Summation over an infinite range
☐	☐	☐	Numeric summation
det (mat1)	25.74	☐	Matrix determinant
det (mat2)	49.42	☐	Matrix determinant
mat1^{-1}	☐	☐	Matrix inverse
mat2^{-1}	167.95	☐	Matrix inverse
mat1^{T}	0.68	☐	Matrix transpose
mat2^{T}	1.41	☐	Matrix transpose
eigVl (mat2)	98.11	☐	Eigenvalues (numeric)
Eigenval (symbmat1)	36.95	☐	Eigenvalues (symbolic)
☐	☐	☐	LU decomposition (numeric)
☐	☐	☐	QR decomposition (numeric)
☐	☐	☐	Cholesky decomposition
simult (mat2, vec1)	☐	☐	Linear system of equations
☐	☐	☐	Linear system of equations $-$ multiple right-hand sides
rref (mat1)	☐	☐	Row-reduced echelon form
☐	☐	☐	Row-reduced echelon form
e^{mat2}	824.53	☐	Matrix exponential (should perhaps try diagonal flattening)

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mat1^{10}	□	□	Matrix power (should perhaps try binary splitting)
mat2^{10}	237.21	□	Matrix power
mat2$^{\pi}$	□	□	Matrix power
SVD (mat3)	□	□	Singular value decomposition
PInverse (mat3)	□	□	Pseudoinverse
mat2 $*$ mat2	□	□	Matrix-matrix dot product
mat2 $*$ vec1	□	□	Matrix-vector dot product
□	□	□	Vector-vector dot product
□	□	□	Matrix norm (Frobenius)
FFT (list2)	22.68	□	Fast Fourier Transform
ChinRem ({{111, 853}, {4293, 283}})	0.80	□	Chinese remainder for integers
□	□	□	Extended gcd for integers
PowerMod (129 140 163, 488 281 255, 7)	0.03	0.11	Integer exponentiation modulo a prime
MultMod (123 456 789, 987 654 321, 7)	0.01	0.08	Integer multiplication modulo a prime
PowerMod (32 361, −1, 113)	0.02	0.13	Modular inverse
NxtPrime (167 168 452 281)	3.03	2.35	Next prime number
Totient (817 723 175 441)	4.05	1.31	Euler totient function
Lagrange (mat5, x)	3.34	□	Lagrange interpolation
mean (list1)	0.09	□	Mean of a list
median (list1)	0.19	□	Median of a list
stdDev (list1)	0.30	□	Standard deviation of a list
□	□	□	Linear regression
Gamma (211 / 2)	1.57	□	Gamma function − half-integer arguments (exact)
Gamma (11.11)	1.03	0.03	Gamma function − real arguments
Gamma (−3 + 2 i)	5.03	0.53	Gamma function − complex arguments
Psi (0, 71 / 2)	0.89	□	Polygamma function − digamma (exact)
Psi (0, 31 / 7)	2.65	□	Polygamma function − digamma (exact)
Psi (0, i)	1.47	□	Polygamma function − digamma
Psi (0, −111.11)	1.05	0.04	Polygamma function − digamma
Psi (13, 2)	1.24	1.05	Polygamma function (exact)
Psi (11, 11.11)	19.00	□	Polygamma function
nCr (1000, 453)	4.34	15.63	Binomial coefficient function
nCr (100 000, −3333)	□	□	Binomial coefficient function
BernNum (50)	6.46	14.74	Bernoulli numbers (exact)
BesselJ (51 / 2, x)	5.79	□	BesselJ function − half-integer order
BesselJ (6, 27.3)	3.79	□	BesselJ function − integer order
BesselJ (111 − 222 i, 0)	0.24	□	BesselJ function
ChebyT (123, 456)	2.32	□	ChebyshevT polynomials − integer order (exact)
ChebyT (22, x)	8.05	0.42	ChebyshevT polynomials − integer order
ChebyT (2.1, 3.2)	0.27	□	ChebyshevT polynomials
Hermite (77, x)	8.89	6.42	Hermite polynomials − integer order
Hermite (100, 1)	15.95	13.40	Hermite polynomials − integer order (exact)

Hermite (3 / 4, 0.1)	20.79	□	Hermite polynomials − fractional order
□	□	□	Hermite polynomials
Legendre (55, x)	6.35	16.17	Legendre polynomial (first kind)
□	□	□	Laguerre polynomials
□	□	□	Spherical harmonics

The following are computed separately and are not included in the timings, since they are just inputs.

Symbol	Value
num1	$3 * 11 * 6\,214\,223 * 337\,636\,146\,415\,297\,021$
list1	seq (rand (), i, 1, 40)
list2	seq (rand (), i, 1, 512)
mat1	randMat (15, 15)
mat2	seq (seq (rand (), j, 1, 20), i, 1, 20)
mat3	seq (seq (rand (), j, 1, 20), i, 1, 10)
mat4	seq ({i, prime (i)}, i, 1, 20)
mat5	{{2, 9}, {4, 833}, {6, 7129}, {8, 31\,233}, {10, 97\,001}, {12, 243\,649}}
symbmat1	diag (seq (expr (string (x) & string (i)), i, 1, 15))
vec1	seq ({rand ()}, i, 1, 20)
poly1	expand $\left(3 * x * (5\,x - 7)^2 * \left(x^2 - 3\,x + 1\right) * \left(x^4 + x - 5\right)\right)$
poly2	expand $\left((3 * x^7 + y) * \left(x - y^2 - 1\right)\right)$
poly3	$4\,x^9 + 21\,x^6 + 2\,x^3 - 3\,x^2 - 3$
poly4	$k^4 - 3\,k^3 + 5\,k^2 - 8\,k + 7$
poly5	expand $\left(\prod (x - i, i, 1, 10)\right)$
poly6	$x^8 - x^7 - 9 * x^6 + 3 * x^5 + 21 * x^4 - 3 * x^3 - 19 * x^2 + x + 6$
poly7	$12\,x^3 - 28\,x^2 + 20\,x - 4$
poly8	$-12\,x^2 + 10\,x - 2$
polysys	$2 * x^2 + x * y - y + 1 = 0$ and $-3 * x * y - x + 2 * y^2 - 2 = 0$ and $2 * x^2 - 3 * x * y^2 + 2 * y^3 - 3 * y + 1 = 0$
ratfunc1	expand $\left(\left((x + y)^2 - (z + 1)^2\right)^n \big/ (x + y + z + 1)^{2\,n} \,\big|\, n = 3\right)$
ode1	$x \cdot y^2 \cdot (y')^2 - 2 \cdot y^3 \cdot y' + 2 \cdot x \cdot y^2 = x^3$
initcond1	y (0) = 1 and y' (0) = 0
boundcond1	y (2) = 3 and y (5) = 2

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Graphics performance

■ **The 68k has the following 3D function plots:**

Wireframe

HiddenSurface

ContourLevels (2D contour plot)

WireAndContour

ImplicitPlot (2D implicit plot)

■ **The 68k has the following differential equation plots:**

SlopeField

DirectionField (can select axes from all dependent and independent variables)

FieldOff (solution plot; can select axes from all dependent and independent variables)

■ **Sequence plots on the 68k:**

Can have time, web, or custom axes

Can select axes from all dependent and independent variables

■ **Conic plots:**

The 49G+ has conic plots, but as far as I can tell, these are trivially replicated on the 68k by solving for y or z as appropriate (or doing a 2D implicit plot)

■ **Note to myself: Include plot settings, such as window settings (typically standard V200 settings)**

Example	TI	HP	Comments
$\sin(x) \cdot \cos(x),\ \{x, -2\pi, 2\pi\}$	☐	☐	2 D function plots − simple function
$\text{floor}(x),\ \{x, -4, 4\}$	☐	☐	2 D function plots − piecewise function
$\text{mod}(\text{ceiling}(x), 3),\ \{x, 0, 10\}$	☐	☐	2 D function plots − piecewise function
$\min(\sin(x), \cos(x)),\ \{x, -3\pi/2, 3\pi/2\}$	☐	☐	2 D function plots − piecewise function
$\text{when}\left(x > 7/3,\ x^{1/3},\ \sin(x^5)\right),\ \{x, -2, 2\}$	☐	☐	2 D function plots − piecewise function
piecewis1, $\{x, -3, 3\}$	☐	☐	2 D function plots − piecewise function
$\left(x^2 - x + 1\right)/\left((x-1)*x^2\right)$	☐	☐	2 D function plots − poles
$\tan(x),\ \{x, -5, 5\}$	☐	☐	2 D function plots − simple poles
$x^2 \text{ when}\left(\sin(x^3) > 0, 1, 0\right),\ \{x, 0, 10\}$	☐	☐	2 D function plots − many discontinuities
$\sin(1/x),\ \{x, -1, 1\}$	☐	☐	2 D function plots − singularity
☐	☐	☐	2 D parametric plots
☐	☐	☐	2 D polar plots
☐	☐	☐	2 D sequence plots
☐	☐	☐	2 D implicit plots
expr1, $\{x, -3, 3\}, \{y, -3, 3\}$	☐	☐	2 D contour plots
$\frac{y}{x^2+y^2+1},\ \{x, -5, 5\}, \{y, -5, 5\}$	☐	☐	3 D function plots
piecewis2, $\{x, -3, 3\}, \{y, -3, 3\}$	☐	☐	3 D function plots − piecewise function
$\text{real}\left(\tan^{-1}(x + i\, y)\right),\ \{x, -\pi, \pi\}, \{y, -\pi, \pi\}$	☐	☐	3 D function plots − discontinuities
$\sin(100\, x\, y),\ \{x, -\pi, \pi\}, \{y, -\pi, \pi\}$	☐	☐	3 D function plots − high frequencies
☐	☐	☐	3 D surface rotation
☐	☐	☐	Differential equation plots
☐	☐	☐	Statistics scatter plots
☐	☐	☐	Statistics bar chart plots
hist1	☐	☐	Statistics histogram plots

Symbol	Value
piecewis1	$\text{when}(x < -1, -1, \text{when}(-1 \le x \le 1, x, \text{when}(x > 1, 1, \text{undef})))$
piecewis2	$\text{when}\left(x^2 + y^2 \le 2,\ x^2 + y^2 + 2,\ \sin\left((x^2 + y^2)/2\right)\right)$
expr1	$\frac{1}{3}\, e^{-(y+1)^2 - x\,(x+2)} \left(9\, e^{2\,x}\,(x-1)^2 + e^{2\,y}\left(6\, e^{2\,x+1}\left(5\,y^5 + 5\,x^3 - x\right) - 1\right)\right)$
hist1	$\text{seq}(\text{randNorm}(2.2, 0.71), i, 1, 50)$

Display routines

The display routines on the 68k are considered rather poor for large expressions. Having many large expressions in the history can slow the home screen down a lot, and often influences computation time.

Example	TI	HP	Comments
299 !	9	□	Big integers
seq (seq (rand (), j, 1, 40), i, 1, 40)	17	□	Large matrices
randMat (40, 40)	7	□	Large matrices
□	□	□	□
□	□	□	□
□	□	□	□

Functionality

■ Compare:

Systems of nonlinear polynomial equations (check how the numerical solvers handle discontinuities)

Laurent series

Asymptotic series

Limits

Hypergeometric summation

Definite integration (check whether 49G+ detects singularities, e.g. $\int (1/(\sin(x)+2),x,0,2\pi)$;compare nInt(1/(x+1),x,-3,3), which gives a "Questionable accuracy" warning on the 68k)

Graphics (e.g. 68k ComplexMap vs 49G GridMap, vector plot, vector field plot, 3D parametric curves and surfaces)

Functionality	TI	HP	Comments
Converting to various trig / exp / hyperbolic forms	□	✓	The 49 G has a lot of expression rewriting tools
Cubic and quartic equations	✓	□	□
Nth-order derivatives	✓	□	□
Implicit differentiation	✓	□	□
Limits of expressions involving factorials	✓	□	Example : $\lim (x! - x, x, \infty)$
Univariate inequality solving	✓	□	Could use Groebner basis techniques for polynomial systems of inequalities

Polynomial roots modulo a prime	✓	☐	☐
Quadratic diophantine equations	✓	☐	Would be nice to be able to handle Thue equations
Polynomial factorization over the integers	☐	✓	Ability to recognize cyclotomic polynomials, factor expand $((x^3 + x + 1)(x^3 - x + 1))$
Squarefree factorization	✓	☐	☐
Equation solving with units	✓	☐	☐
Continuous integrals for continuous integrands	✓	✓	Example : $\int \frac{1}{2\cos(x)+3}\, dx$
Rothstein-Trager algorithm	✓	☐	Example : $\int \frac{6x^5+6x^4-8x^3-18x^2+8x+8}{x^6-5x^4-8x^3-2x^2+2x+1}\, dx$
Risch integration	☐	✓	I attempted a (nearly useless) Risch implementation for the TI-68k
Rational function integration (high-degree)	✓	☐	Example : $\int \left(1/\left(x^4+x^3+x^2+x+1\right)\right) dx$
Heuristic integration including table lookup	✓	☐	Useful for speed and for cases not covered by algorithms
Hyperelliptic integrals	✓	☐	Example : $\int \left(\frac{3x^5-x+2}{\sqrt{x^5+x-1}\,(x^5-x^2+x-1)}\right) dx$
Subresultants	✓	☐	☐
Reduction of a polynomial wrt other polynomials	✓	☐	"Simplification" with side relations
Heuristic GCD	☐	✓	GCDHEU on the 49 G, I believe
Solving simple difference equations	✓	☐	Similar to solving differential equations
Integral transforms and inverses	✓	☐	Fourier, Laplace (Hilbert, Mellin, Hankel not done yet)
Z-transform and inverse	✓	☐	☐
Multidimensional definite integrals	✓	☐	☐
Complex residues	✓	☐	Would be helpful for contour integration
Symbolic Laurent series	☐	✓	Including asymptotic series
Symbolic products	✓	☐	☐
Pade approximant	✓	☐	☐
Series arithmetic and series composition	☐	☐	☐
Zero-equivalence testing	☐	☐	☐
Simplification	☐	☐	Example : $\sqrt{2\cdot\sqrt{3}+4} \ \to\ \sqrt{3}+1$
Vector analysis in general coordinate systems	✓	☐	☐
Quine-McCluskey algorithm	✓	☐	Minimal SOP or POS expression from minterm or maxterm lists
Tensor analysis	✓	☐	☐
Optimization	✓	☐	Linear programming, Lagrange multipliers
Symbolic solution of ODEs and PDEs	✓	☐	Linear first-order homogeneous ODE systems, linear high-order constant-coefficient (non) homogeneous ODEs, linear constant-coefficient PDEs, quasilinear first-order PDEs, separation of variables, etc.
5 th-order Runge-Kutta for numerical ODE solving	✓	☐	☐
Numerical ODE boundary value problem solving	☐	☐	☐

Symbolic solution of systems of ODEs	✓	□	Lars Frederiksen's DiffEq package can do this
Integral equations	✓	□	Lars Frederiksen's Advanced Laplace package can do this
Statistics	✓	□	distributions / inverses, random numbers, regressions, hypothesis tests, (M) Anova, confidence intervals
Nonlinear regression for arbitrary model function	□	□	SVD could be useful here
Continued fractions	✓	□	□
Binomial coefficients with negative numbers	✓	□	Examples : nCr (−2, 3), nCr (3, −2)
Arbitrary-precision arithmetic	□	✓	There is a GMP demo for the TI-68k
Matrix operations modulo a prime	✓	□	Determinant, inverse, and nullspace (need to add power)
Solving singular linear systems	✓	□	TI's SimultEq Flash app
Kronecker product of matrices	✓	□	□
Gram-Schmidt orthonormalization	✓	□	□
Schur, LQ matrix decompositions	□	✓	□
Symbolic matrix decompositions	✓	□	□
Sparse matrices	□	□	MathTools has very limited sparse matrix support
Finding a linear recurrence for data	✓	□	□
Series convergence acceleration	✓	□	Aitken δ^2, Euler-Maclaurin
Radix conversions for fractional parts	✓	□	□
Permutations, subsets, etc. of a list	✓	□	□
Prime generation	✓	□	Next / previous prime, nth prime, random prime in interval
Chinese remaindering	✓	□	Solving more than two congruences simultaneously
Lattice reduction using LLL	✓	□	Can be useful for factor recombination (polynomial factoring)
PSLQ integer relation algorithm	□	✓	□
Algebraic number arithmetic	□	□	There is some basic stuff for this in MathTools
Graph editing and drawing	□	✓	With additional routines to find shortest or cheapest path, do various sorts, etc.
Geometry drawing and transformations	✓	□	Sketchpad, Cabri
Import / export of standard data / graphics formats	✓	□	Excel spreadsheet, MathML, XML, etc.
Y-slice plots	□	✓	Animated plot of z (x, y) vs y for changing values of x
Statistics xy-line and modified box plots	✓	□	□

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